Fall 2014-15

EE 531 MT2 Part – 2

Take-Home Problem

Due: Jan 20, 2015, 17:30

<u>Problem Description</u>

In some target detection applications, a target is declared as detected if it is detected M times out of N looks $(M < N)$. For example, the 2-out-of-4 detection rule delays the final decision on the presence or absence of the target until its second detection in the following three looks. A state transition diagram of 2-out-of-4 detection rule is given below.

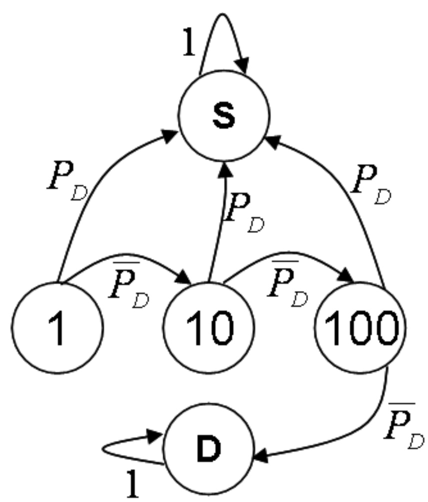

Fig. 1: State transition diagram for 2-out-of-4 detection rule

Here, P_D is the target detection probability and $\overline{P}_D = 1 - P_D$ is the probability of the complementary event. States **S** and **D** correspond to successfully detected target and dropped target states, respectively. The state **1** corresponds to the initial detection of the target and it is the initial state of the chain, in general.

In this problem, we evaluate the probability of reaching state **S** given that the initial state is **1** $(X_0 = 1)$; the expected number of steps to reach an absorbing state given that $X_0 = 1$ etc.

Take-Home Problem:

1. Draw a state transition diagram for the 3-out-of-5 decision rule. (The rest of take-home exam problem is on the 3-out-of-5 decision rule.)

2. Calculate the probability of reaching state **S**, given that the initial state is **1**, for $P_D = \{0, 0.01, 0.02, \ldots, 0.98, 0.99, 1\}$. (Numerically evaluate the probability of this event using the theory of Markov chains discussed in the lectures.) Plot the event probability vs. P_D.

3. Verify the results of part 2 using Monte Carlo simulations for $P_D = \{0.1, 0.2, \ldots, 0.9\}$. Plot the Monte Carlo estimate for the event probability on the same figure given in part 2.

   (To implement the Monte Carlo simulations, you initiate the chain at state **1** and decide on the next state by generating a *realization* of the Bernoulli random variable with probability P_D (biased coin-toss). Hence with the probability P_D, you go to a specific state and with probability $\overline{P_D}$ you go to another. You repeat this process until reaching one of the absorbing states. The event probability of interest is approximated by counting the number of times that you reach state **S** for a given number of Monte Carlo trials. For example, if you reach state **S,** 50 times in 1000 trials; the event probability is $\approx 50/1000$. With the law of large numbers, we expect the Monte Carlo estimate to converge to the true value as the number of Monte Carlo trials increases.)

4. Evaluate the expected number of steps until reaching states **S** or **D,** given that $X_0 = 1$, for $P_D = \{0, 0.01, 0.02, \ldots, 0.98, 0.99, 1\}$. Verify your results using Monte Carlo simulations for $P_D = \{0.1, 0.2, \ldots, 0.9\}$. Plot the expectation results and the Monte Carlo estimates in the same figure.

5. Using Monte Carlo simulations, approximately evaluate the number of steps required to reach state **S** (but not state **D**), given that $X_0 = 1$, for $P_D = \{0.1, 0.2, \ldots, 0.9\}$.

   Bonus: Present an analytical formulation for the expectation result discussed in part 5 and verify your analytical result via Monte Carlo simulations.

Notes: 1. Late submissions are not allowed (this is an exam!).
2. Submit *hard-copy* of your results with relevant explanations along with the Matlab code. (Do not email your results to me, this is an exam!)
3. You can submit your results no later than **Jan 20, 2015, 17:30** to my office.